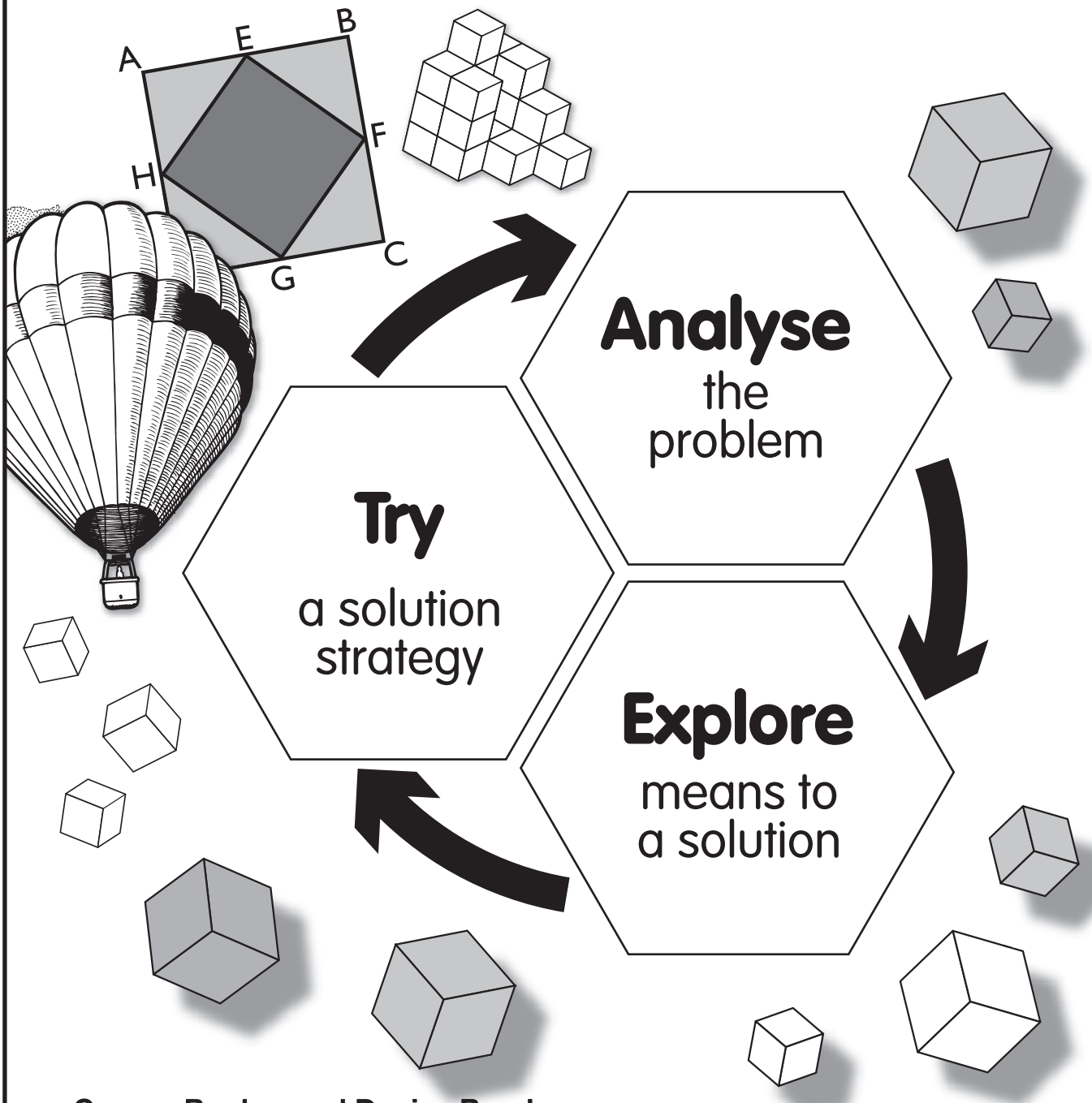
PRIMARY

BOOK E
(Ages 9–10)

CURRICULUM LINKED

Problem-solving in mathematics



George Booker and Denise Bond

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SAMPLE- NOT FOR SALE

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SAMPLE- NOT FOR SALE

Problem-solving
To use spatial visualisation and logical reasoning to solve problems.

Curriculum links
Fourth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Represent mathematical ideas and processes in different modes: pictorial and diagrammatic
- Explore and investigate mathematical patterns and relationships

Shape and space
- Solve and complete practical tasks and problems involving 3-D shapes

Materials
dice, cubes, grid paper to draw views of the shapes and nets of cubes, digital camera to photograph the shapes

Focus
These pages explore arrangements and dissections of three-dimensional shapes in order to determine how particular outcomes are formed. Spatial and logical thinking and organisation are required as pupils investigate all likely arrangements to ensure the final shapes match the given criteria or visualise a shape in terms of its component parts.

Discussion
Page 3
This page requires an ability to visualise the arrangements of cubes stacked in several layers. Some pupils will see layers from the bottom or top, while others will see slices going across the shape. They then need to use this information to determine where paint is applied to the complete shape and visualise the effect on the individual cubes. Some pupils may need to build the structure and manipulate the blocks in order to see what can happen. Encourage systematic analysis when they are working out the results. Only certain faces of the cubes are able to be painted and the number of cubes counted must equal the number of cubes in the shape.

Page 4
This investigation encourages pupils to see three-dimensional shapes in terms of the component two-dimensional forms. When viewed from above and below, the outline will appear the same. The views from opposite sides will be mirror images of each other. Understanding this can be difficult and pupils need to be encouraged to be systematic in constructing the shapes. When a pupil makes a shape for another pupil, rather than draw the different views, he or she may be able to take digital photos of each and then print them for other pupils to build from.

Page 5
The activities on this page ask pupils to visualise how a two-dimensional representation can be folded to make a cube. Some pupils may need to cut out the nets from grid paper and fold them to see which form cubes. The dice at the top of the page shows 6, 3 and 2 dots, so the opposite sides would have 1, 4 and 5 dots. (It is important to realise that the opposing numbers on a dice always add to seven.) Seeing which dots would occur on the blank squares requires pupils to visualise how the squares fold and which faces would be opposite to each other. Again, some pupils may need to make the nets to see or check results. There are several ways the digits could be arranged on the blank die, with the opposite sides adding to seven. Writing letters on the net to match the given views extends this thinking.

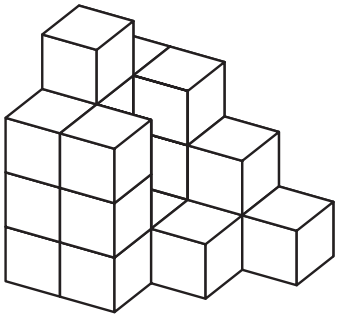
Possible difficulties
- Unable to visualise the shapes from the two-dimensional representations
- Only considers the cubes that can be readily seen on the outside of the shapes
- Unable to keep track of the possibilities in any of the investigations
- Unable to visualise the opposite sides from the net of the cube

Extension
- Pupils make stacks using different arrangements of cubes and ask other pupils to investigate what would happen if all of the outside faces were painted.
- Have pupils write their own name (if it has six letters) or a six-letter word on a cube and then challenge a partner to draw a net of the cube.
- Ask pupils to write a six-letter word on a net and then challenge a partner to draw them on a representation of the cube.

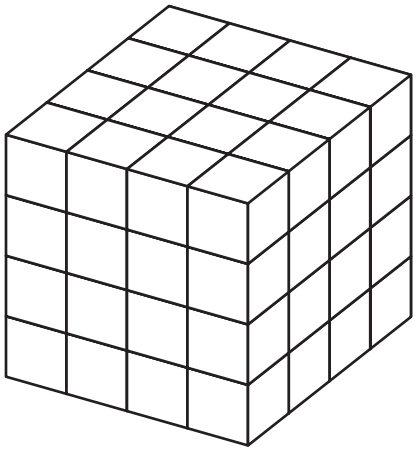
SAMPLE- NOT FOR SALE

PAINTING CUBES

Some cubes were joined together to make this shape. After they were joined together, the shape was painted yellow on all of its sides. When it was taken apart, some faces of the cubes were painted yellow and some weren't.



1. How many cubes are there in the shape? _____
2. Make a table to show how many cubes have 0, 1, 2, 3, 4 or 5 faces painted.
3. Have you included all of the cubes? _____

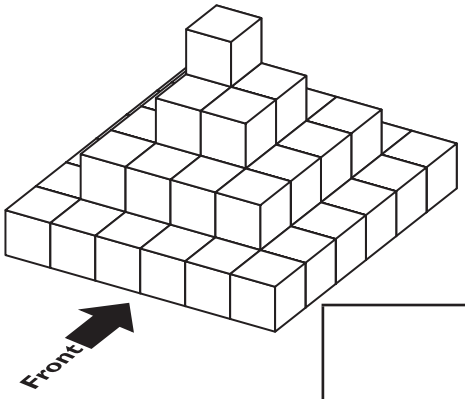


A wooden cube is painted so that two opposite sides are green, another two opposite sides are yellow and the last two sides are red. The cube is then cut into 64 small cubes.

4. How many of the smaller cubes will have one face painted green, one face painted yellow and one face painted red? _____
5. (a) How many of these cubes will have only two faces painted? _____
   (b) How many are red and green? _____
6. (a) How many of the cubes will have only one face painted? _____
   (b) How many are yellow? _____
7. How many of the cubes would have no paint? _____
8. Are there any cubes painted in other ways? _____

SAMPLE- NOT FOR SALE

VIEWING CUBES



Draw what this shape would look like viewed from above. Would it look any different if viewed from below?

1. (a) Draw it from each of the sides in the space provided.

	Top	Bottom	
Left	Front	Back	Right

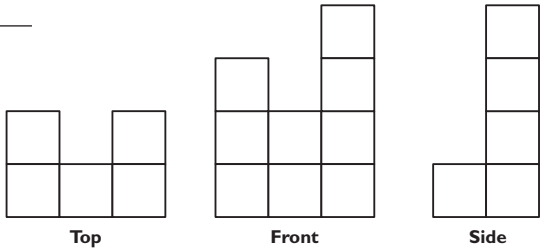
(b) How do the views compare?

_____

Get some cubes and make the shape.

2. How many cubes did you need? _____

These are the views of an arrangement of cubes from the top, front and side.



Get some cubes and make the shape.

3. How many cubes did you need? _____

4. Make and draw a shape of your own. Ask a friend to construct the shape using your drawings. Compare the result to your drawings.

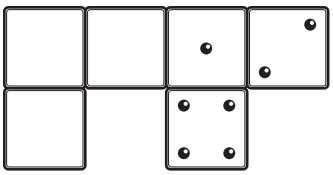
SAMPLE- NOT FOR SALE

NETS AND CUBES

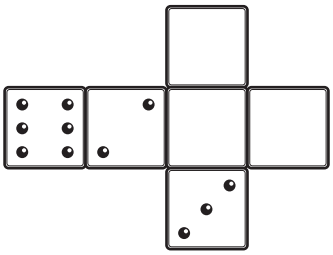
Most dice are made in the shape of a cube, with the dots on each square face arranged so the sum of the dots on opposite faces is always seven. An arrangement of squares that can be folded to make a cube is called the net of the cube.

1. Which of the arrangements of squares form nets for dice? Circle the ones that form a correct net and draw the missing dots on the blank squares.

(a)



(b)



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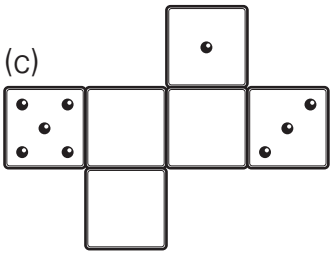

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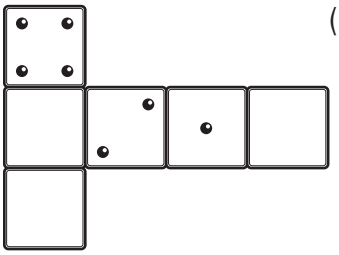

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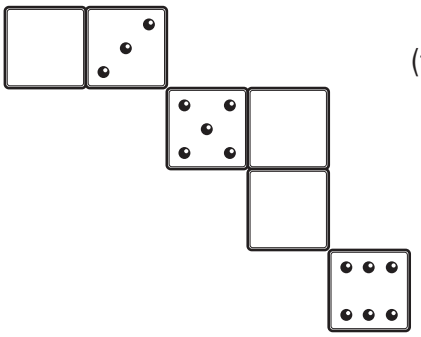

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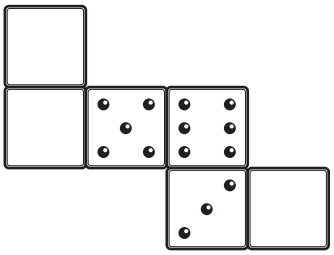
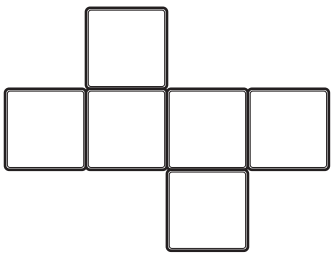
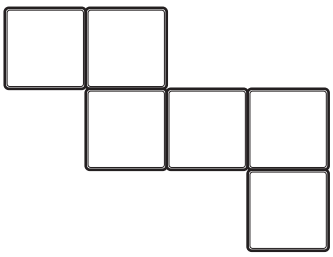
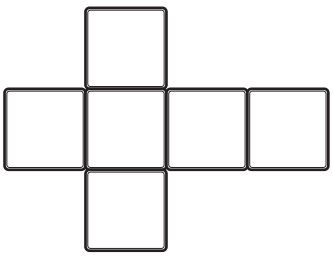


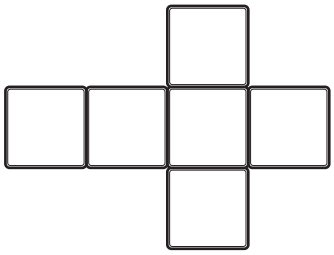
2. Put digits on the squares on the nets to make dice. Make sure the sum of the digits on the opposite faces is seven.







3. Here are three views of a dice on which Lauren wrote one letter of her name on each face. Can you draw a net to show how the letters would be arranged on the squares?



SAMPLE- NOT FOR SALE

Problem-solving
To solve problems involving time and make decisions based on particular criteria.

Curriculum links
Fourth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Devise and use mental strategies and procedures for carrying out mathematical tasks

Measures
- Read and interpret simple timetables
- Solve and complete problems involving times

Materials
clock

Focus
This page explores reading for information from a number of sources (information about the plane, the timetable, and the shuttle bus) and using it to find solutions. The problems involve thinking about and working with time. Decisions about time being 'too early' or 'too late' are considered rather than an exact time.

Discussion
Page 7
Pupils read the information on the page and use it to solve a number of questions. Pupils who are not familiar with 24-hour time can complete the activity by using a conversion table. The worksheet can be used to introduce the concept of 24-hour time. Pupils need to read and sort information from a number of sources and fit this information against set criteria.

There are a number of flights that fit the criteria, with other flights arriving at Barcelona being either 'too early' or 'too late'. Once a flight is deemed to be too late, then others that are later still can be automatically excluded; for example, the Lion Airlines plane at 16:15 does not arrive at Barcelona until after dinner. This then also rules out the flights at 17:30 and 18:00.

Similar thinking can be used for the flights that are too early. For example, catching the Lion Airlines plane at 09:50 would get to you to the hotel at around midday, which is about two hours too early. As such, the flights at 07:45, 08:10 and 09:15 are also too early and can be excluded.

Pupils need to think in terms of 24-hour time for the flight information but the before and after times of 2 pm and 6 pm are in 12-hour time. Pupils need to be able to convert the times to 24-hour time.

The information regarding the taxi and the waiting time at the airport is additional information which is not needed. Some pupils may try to include this in their calculations. The bus leaves the airport every 30 minutes (15 minutes and 45 minutes past each hour).

Time between the plane arriving and bus leaving also needs to be considered. If the plane arrives at 10:10 it would not be possible to catch the 10:15 bus as you would need to leave the plane, wait for luggage and then find the bus and you would need longer than 5 minutes to do this.

Possible difficulties
- Unfamiliarity with a timetable
- Confusion with 24-hour time
- Not excluding the taxi and wait time information
- Thinking that an exact flight is needed rather than flights that are neither too early or too late

Extension
- Use the information and timetable with other criteria; for example, if you need to be in Barcelona for a lunchtime meeting what flights can you take?

SAMPLE- NOT FOR SALE

HOLIDAY FLIGHTS

Imagine you have decided to travel from Dublin to Barcelona for a holiday.

Look at the following information and plan your holiday. As most hotels have a 2 pm check-in time, you want to arrive after 2 pm. However, you also want to arrive before dinner at 6 pm.

TAXI INFORMATION

from home to Dublin Airport

- waiting time 25 mins
- travel time 45 mins

PLANE INFORMATION

- wait time at airport 1 hour
- flight time to Barcelona 1 hour 50 mins

Star Airlines

Departure times	Flight numbers					
	ST23	ST32	ST47	ST51	ST68	ST74
	07:45	09:15	12:40	13:50	15:00	17:30

Lion Airlines

Departure times	Flight numbers					
	LA23	LA32	LA47	LA51	LA68	LA74
	08:10	09:50	14:30	15:10	16:15	18:00

BUS INFORMATION

from Barcelona Airport to hotel

- hotel bus leaves airport every 30 mins (15 and 45 mins past each hour)
- travel time to hotel 15 mins

1. I can catch the following flights: _____
2. These flights are too early: _____
3. These flights are too late: _____

SAMPLE- NOT FOR SALE

Problem-solving
To read, interpret and analyse information.

Curriculum links
Fourth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Devise and use mental strategies and procedures for carrying out mathematical tasks

Number
- Explore and identify place value in numbers
- Solve word problems involving addition and subtraction
- Solve and complete problems involving multiplication and division of whole numbers

Materials
calculator, number expanders

Focus
These pages explore concepts of place value and number sense. The relationships among numbers and place value is analysed and pupils are encouraged to not only find possibilities but to also disregard numbers and combinations that are not possible. Place value and number sense are needed rather than addition or multiplication.

Discussion
Page 9
This investigation requires pupils to read and interpret information while thinking in terms of place value. As the book has 10 chapters of equal length, an understanding of place value can be used to solve each problem. Given that the book has 1380 pages, we know from place value that this number has 138 tens so each chapter must be 138 pages long. This information can be used to calculate the various problems. Before starting, pupils may find it helpful to draw up a table outlining the starting page of each chapter. Each chapter has 138 pages and Chapter 1 starts on page 1, so Chapter 2 must start on page 139 and so on. The page number that Wanda has read to is given in the beginning and this information is needed to answer some of the problems. For example, in order to determine how many pages Wanda has read past the middle of the book, it is necessary to keep in mind that there are 1380 pages in the book and she has read up to page 759. Similarly, this information is again needed to determine how many pages need to be read to finish the book.

Page 10
These problems require pupils to use their calculator to generate some results in order to think about the underlying numeration and multiplication patterns. For Problem 1, each time a three-digit number, such as 345, is changed to a six-digit number (345 345), it is at first surprising that it can be divided exactly by 13. The result also divides exactly by 11 and this result divides exactly by 7 to give the initial three-digit number. Reading the number as three hundred and forty-five thousand, three hundred and forty-five gives a clue—the original number is multiplied by 1000 and added to itself or multiplied by 1001. The product of 13 x 11 x 7 is 1001, so the divisions have undone the effect of the multiplying.

Problem 2 requires pupils to notice a pattern: That the product of the two increased and decreased numbers is the square of the original number, subtracted the square of the number that was originally added and subtracted. Using the calculator's memory to store the square and then subtract the product of the two changed numbers allows this pattern to be readily seen. It could be expressed as:
(number + increase) x (number – increase)
= number^2 – increase^2.
This leads to a powerful and well-known algebraic identity.

Page 11
In this investigation pupils need to read and interpret the information and use it to find combinations that match specific criteria. Pupils need to think of possible combinations as well as discarding combinations that don't work. In Problem 1, each person needs to receive 15 caramel toffees. Possible combinations are boxes 9, 5, 1; or 8, 4, 3; or 7, 6, 2. Problem 2 has many possible combinations. The two children can be given boxes that total to 18 or 20 caramel toffees and each total has several possible combinations; for example, they could have boxes 2, 3, 5, 8 and 1, 4, 6, 7 (if totalling to 18); or 1, 4, 6, 9 and 2, 3, 7, 8 (if totalling to 20). The last question asks for a total of 13 toffees, so possible combinations include 9, 3, 1 and 7, 4, 2 or 2, 3, 8 and 1, 5, 7.

Possible difficulties
- Poor understanding of place value
- Wanting to add, subtract or multiply rather than using place value or number sense
- Not using all of the criteria
- Not using place value to solve the questions
- Not taking into consideration the starting page of each chapter

Extension
- Change the criteria involving the number of pages in a book and the page number read to and explore the problems again based on the new criteria.
- Pupils could think up their own calculator problems and write the criteria to match.
- Work out the different possibilities for two children to have three boxes, each with different totals.

SAMPLE- NOT FOR SALE

BOOKWORMS

Wanda's book starts on page 1 and has 1380 pages. There are 10 chapters of equal length in the book and she has read up to page 759.

1. How many pages are in each chapter?

2. Wanda's favourite page is 409. What chapter is it in?

3. Wanda reads 10 pages each morning and 10 pages each night. How long has she been reading the book?

4. Wanda's favourite chapter is Chapter 5. What pages are in Chapter 5?

5. The most exciting part was from page 619 to page 694. What chapters are these pages in?

6. How many pages has Wanda read past the middle of the book?

7. Wanda's friend is also reading this book. She has read eight pages of Chapter 6. What page is she up to?

8. How many more pages does Wanda's friend need to read to finish the book?

9. How many more pages does Wanda need to read to finish the book?

10. If Wanda reads 10 pages per day, how long will it take her to read the book?

SAMPLE- NOT FOR SALE

CALCULATOR PATTERNS

Use your calculator to help you think about what happens.

Increasing and dividing

1. Enter a three-digit number (e.g. 345).
   - Repeat the three digits to make a six-digit number (e.g. 345 345).
   - Divide this number by 13.
   - Divide the result by 11.
   - Divide this result by seven.

   (a) What number is this?

   _____

   (b) Try again with another three-digit number. What happens?

   _____

   _____

   (c) Why does this happen?

   _____

   _____

Squaring and multiplying

2. Enter a two-digit number (e.g. 78), square it and store the result in your calculator's memory.
   - Multiply the numbers that are one more and one less than your original two-digit number (e.g. 79 x 77).
   - Compare the answer to the square of your original number.
   - Try some other two-digit numbers.

   What do you notice? _____

3. Use another two-digit number, but now multiply the numbers that are two more and two less.

   What pattern can you see? _____

4. Now try another two-digit number, but multiply the numbers that are three more and three less.

   (a) What is the pattern now? _____

   (b) What do you think will happen if you try four more and four less?

   _____

   (c) What about five more and five less? _____

5. Describe a pattern that will explain what happens whenever you multiply two numbers, each increased and decreased by the same amount from an original number.

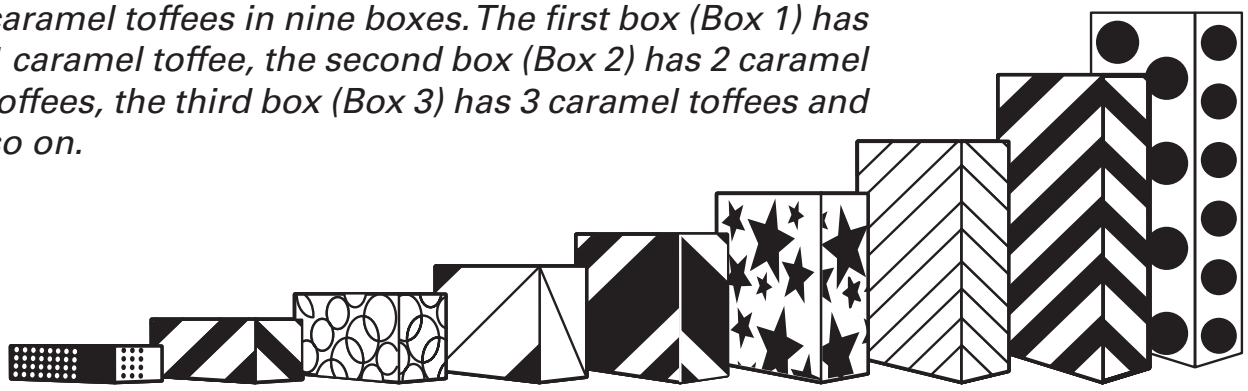
   _____

   _____

SAMPLE- NOT FOR SALE

CARAMEL TOFFEES

Trevor is having a party. He has wrapped an amount of caramel toffees in nine boxes. The first box (Box 1) has 1 caramel toffee, the second box (Box 2) has 2 caramel toffees, the third box (Box 3) has 3 caramel toffees and so on.



Box 1 Box 2 Box 3 Box 4 Box 5 Box 6 Box 7 Box 8 Box 9

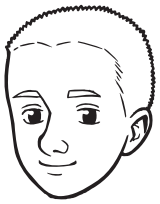
1. How many caramel toffees has Trevor wrapped? _____

Trevor wants to give the same number of caramel toffees to each person.

2. If he gives three children 3 boxes each and an equal amount of toffees to each child, which boxes could he give?

(a)

_____

(b) 

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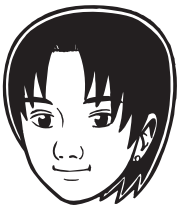
(c) 

_____

3. If he gives two children 4 boxes and an equal amount of toffees each, which boxes can he give?

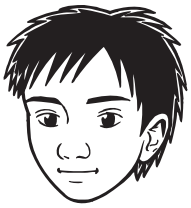
(a)

_____

(b)

_____

4. If he gives two children 3 boxes each, which boxes could he give so each child receives 13 caramel toffees?

(a) 

_____

(b)

_____

SAMPLE- NOT FOR SALE