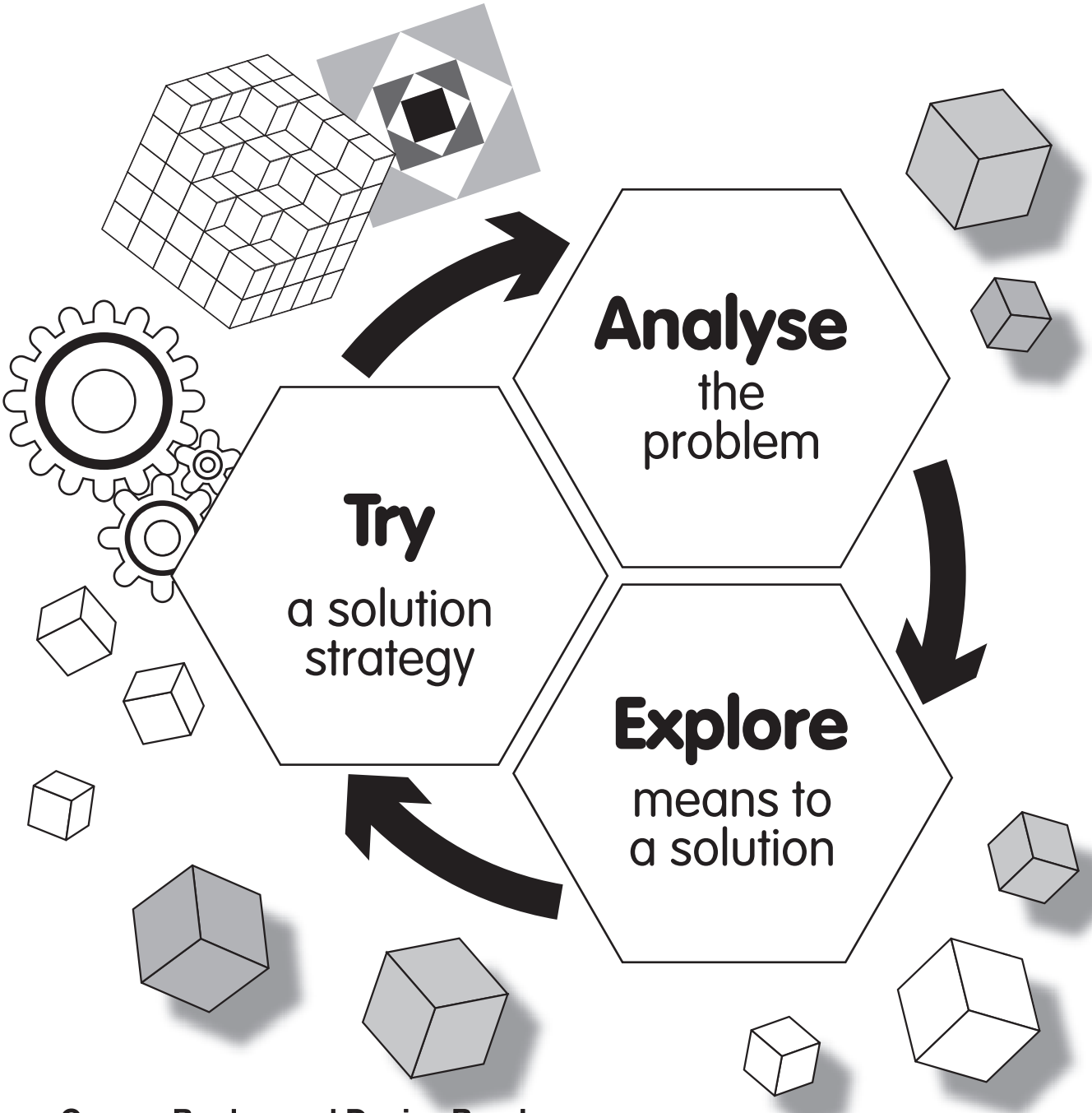
PRIMARY

BOOK G
(Ages 11+)

CURRICULUM LINKED

Problem-solving in mathematics



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SAMPLE - NOT FOR SALE

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Problem-solving

To use spatial visualisation and logical reasoning to solve problems

Curriculum links

Sixth Class

Skills

- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Record the processes and results of work using a variety of methods
- Represent mathematical ideas and processes in different modes
- Search for and investigate mathematical patterns and relationships

Measures

- Measure the surface area of specified 3-D shapes

Materials

cubes, calculator

Focus

These pages explore arrangements of cubes and longer shapes in order to determine surface areas and volumes and to investigate the patterns they exhibit. Spatial as well as logical thinking and organisation are involved as pupils visualise a given shape in terms of its component parts.

Discussion

Page 3

Pupils need to visualise how the items are stacked, determine which surfaces are on the outside and calculate the sum of all the areas. When the builder decided to paint all of the outside faces, there will be an additional ten 2 m x 20 cm rectangles to be painted. Care will be needed to ensure all units used are the same (metres expressed as decimal fractions or centimetres). When the 3 x 3 cube is considered, there will be one 1 cm^3 in the middle—when it is removed to create the hole all the way through the cube, the surface area painted will be 6 cm^2 less. The 4 x 4 cube will have a 2 x 2 cube in the middle—the surface area will be 24 cm^2 less when this is removed to make the hole. There are different possibilities for the 5 x 5 cube according to whether the hole is 1 cm x 1 cm or 3 cm x 3 cm when cubes are removed to make the same hole from all 6 sides. Pupils may need to use cubes to see what is happening.

Page 4

This set of investigations requires visualising the arrangement of the cubes in the inside of the prism and then seeing how there is another layer on each side to give the whole prism.

An understanding of prime factors is needed to determine the inside shape. Stacking cubes to form the inside shapes will then assist pupils to understand what is required in these problems. In the first problem, 105 has prime factors 3 x 5 x 7 which gives the dimensions of the inside, unpainted shape. Another layer on each side shows that the whole prism is 5 x 7 x 9 and the surface area is

(2 x 35) + (2 x 45) + (2 x 63) or 286 cm^2.

363 has prime factors 3 x 11 x 11 and can be solved in a similar manner.

60 has prime factors 2 x 2 x 3 x 5. These need to be considered 3 at a time to give four possible answers:

4 x 3 x 5
2 x 6 x 5
2 x 2 x 15
or 2 x 3 x 10

Page 5

This set of investigations encourages pupils to see 3-dimensional shapes in terms of the component cubes and work out the surface area by determining the number of exposed faces and the volume by working out the total number of cubes. The cubes vary in size so other factors also come into the calculations. Pupils need to work directly from a diagram that shows how the cubes are arranged and from plans that use a grid to indicate the number of cubes in each position. The final investigation requires pupils to make a shape of their own, construct a plan and then give the plan to another pupil who has to determine the surface area and volume of the shape.

Possible difficulties

- Unable to visualise the 3-dimensional shapes from the information given
- Does not realise the internal prisms are given by considering the prime factors of the number of cubes
- Does not take into account the dimension of the different cubes when working out volumes
- Does not consider all of the surfaces when working out surface areas

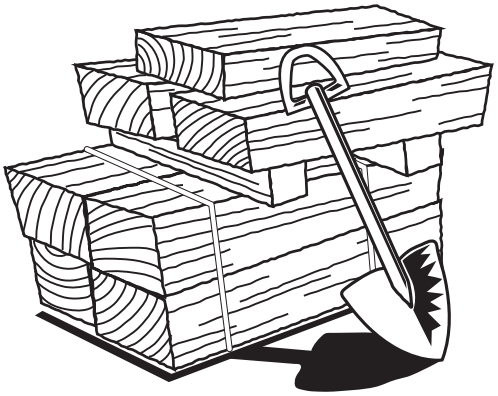
Extension

- Investigate other arrangements of railway sleepers to make steps.
- Reverse the problem with the prisms: if 429 715 or 1001 cubes were used to make a rectangular prism which was then painted yellow on the outside, how many cubes would not be painted? What would be the volume and surface area of the unpainted prism?
- Have pupils make up their own problems like the last problem on page 3; e.g. 36 unpainted cubes of side 3 cm, 48 unpainted cubes of side 2 cm or 64 unpainted cubes of side 4 cm.

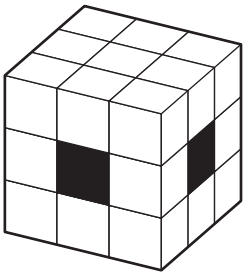
SAMPLE - NOT FOR SALE

SURFACE AREA

1. Some 2 m long railway sleepers with 10 cm x 20 cm rectangular ends are going to be stacked to form a set of garden stairs with 5 steps. If all the exposed sides except those on the bottom and on the higher end set into the garden are painted with wood preserver, what area will be painted?



2. When the builder put the steps together, he decided to paint all the outside surfaces before he assembled the steps in order to prolong the life of the stairs. What area did he actually paint?



3. Imagine 1 cm^3 cubes stacked to make a 3 x 3 x 3 cube. Remove one cube from the centre of each face. If all of the exposed faces are painted red, what is the surface area of the shape that has been painted?

4. Would the surface area to be painted change if there was a hole through the centre of the cube?

5. (a) Now imagine a 4 x 4 x 4 cube. Remove a square of cubes from the centre of each face. What would be the surface area of this shape?

   (b) How would the surface area change if there was a hole in the centre of this cube?

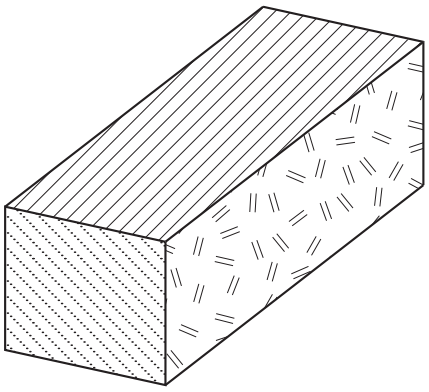
6. If you had a 5 x 5 x 5 cube and removed a square of cubes from the centre of each face, what surface area could there be? (There are 2 possibilities.)

7. How would the surface area change if there was a hole in the centre of the cube? (Consider all the possibilities.)

SAMPLE - NOT FOR SALE

VOLUME AND SURFACE AREA

1. A number of 1 cm^3 cubes are put together to make a rectangular prism with each edge greater than 1 cm. The six faces of the prism are painted green. When the small cubes are taken apart, 105 have no paint on them.



   (a) Where would these cubes occur?

   _____

   _____

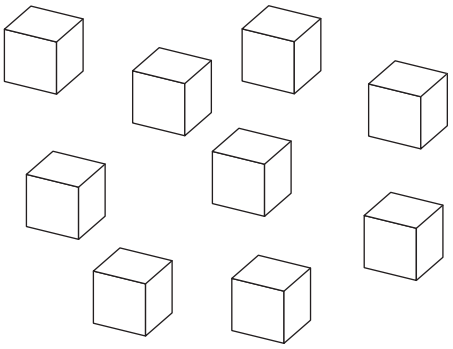
   (b) What shape would they form?

   _____

   Use this information to find:

   (c) the volume of the whole prism _____

   (d) the surface area of the whole prism. _____



2. A large number of 1 cm^3 cubes are put together to make another rectangular prism with edges greater than 1 cm. After the six faces of the prism are painted green and then taken apart, 363 of the small cubes have not been painted.

   What is the volume and surface area of the whole prism?

   _____

   _____

3. Some 2 cm^3 cubes are put together to make a rectangular prism. The six faces of the prism are painted blue. When the small cubes are taken apart, 60 have no paint on them.

   (a) In which ways could 60 cubes form the internal prism?

   _____

   _____

   (b) What would be the volume and surface area of the whole prism for each possibility?

   _____

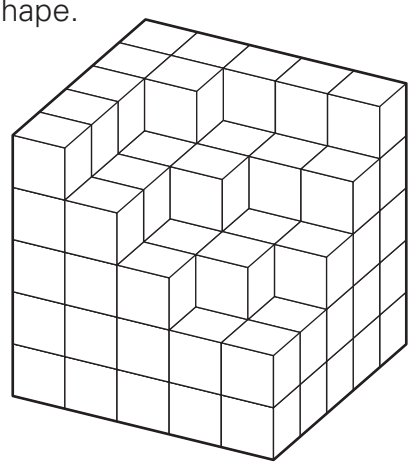
   _____

   _____

SAMPLE - NOT FOR SALE

SURFACE AREA AND VOLUME

1. Some 2 cm^3 cubes have been arranged to make this shape.



(a) What is the surface area of the shape?

_____

(b) What is the volume of the shape?

_____

(c) If you add one more block to the upper surface of every block, what will be the new surface area and volume?

_____

6	6	4	2
7	6	4	
6	5	3	
5	4		

2. Here is a plan of a shape made from a number of 3 cm^3 cubes. The squares show where the cubes are and the numbers show how many cubes at each place.

(a) Get the number of cubes you will need and make the shape. Use this model to work out the surface area and volume of the shape.

_____

(b) Work out the surface area and volume of the shapes shown by these plans. The size of the cubes is written underneath each plan.

(i)

5	4	3	2	1
4	6	6	4	2
3	2	1	1	1

2 cm^3 cubes

(ii)

6	3		
5	3		
5	3		
4	3	2	1
4	3	2	1

3 cm^3 cubes

(iii)

5	3	2	4
3	5	4	2
2	4	5	3
4	2	3	5

4 cm^3 cubes

_____ _____ _____

(c) Can you work out the volume and surface area directly from the plan? (You may need to draw the views from each side and the top to find the surface area.)

(d) Make a shape of your own. Then make a plan for it and decide on the dimensions of the cubes. Ask a friend to work out the surface area and volume. Does your friend need to make the shape?

SAMPLE - NOT FOR SALE

Problem-solving
To use logical reasoning and number sense to solve problems

Curriculum links
Sixth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Record the processes and results of work using a variety of methods
- Represent mathematical ideas and processes in different modes
- Reason systematically in a mathematical context
- Use mental strategies and procedures for carrying out mathematical tasks

Materials
counters

Focus
This page explores problems based on a conceptual understanding of whole numbers and fractions and an understanding of what makes sense in the problem contexts. Backtracking from the final position will assist in understanding and solving the problems but counters could also be used to keep track of what is happening. Using a diagram or calculator are other ways to sort through the information while keeping the intent of the problem in mind.

Discussion
Page 7
These problems highlight the need to carefully analyse the problem before starting on a solution. In the first problem, more than half the avocados are sold on the Saturday if 25 less than half is sold on the Sunday. One way to solve the problem involves 'Try and adjust' using a table to keep track:

	Saturday	Sunday	total sold
Try	300	125	425 – too few
Try	400	175	575 – too many
Try	390	170	560 – too few
Try	398	174	572

A form of algebraic reasoning is another way.
The number sold on Sunday is (half the number sold on Saturday – 25).
The number sold, 572, is the number sold on Saturday + (half the number sold on Saturday – 25).

1.5 x the number sold on Saturday – 25 is 572, so 3 x the number sold on Saturday – 50 is 1144.

3 x the number sold on Saturday is 1194.

398 avocados are sold on Saturday.

In the second problem, clearly Lance is not actually going to sell half a tomato! At each step, there must be an odd number of tomatoes. When he sells these tomatoes, they will both get half of the nearest even number and the customer will get 1 extra tomato. This means that before Lance sold the last tomatoes, the third customer must have taken 56 leaving Lance 55 tomatoes to sell. The customer before that must have taken 112 tomatoes leaving Lance 111 tomatoes to sell. So the first customer took 224 tomatoes leaving Lance 223 tomatoes to sell. Lance started with 447 tomatoes:

Lance	third customer	Lance	second customer	Lance	first customer	Lance
55	56					
		111	112			
				223	224	
						447

The third problem is similar to the second problem but this time the seller must add 3 to the number of pumpkins left to find half of the number he had to sell. Counters, a diagram similar to Problem 2 or a table for 'Try and adjust' show this readily.

Try	Lance	First	Lance	Second	Lance	Third	Lance/last
50	22	27	8	14	1	7	too few
60	27	33	X				
58	26	32	10	16	2	13	too few
62	28	34	11	17	X		
64	29		X				
66	30	36	12	18	3	9	3

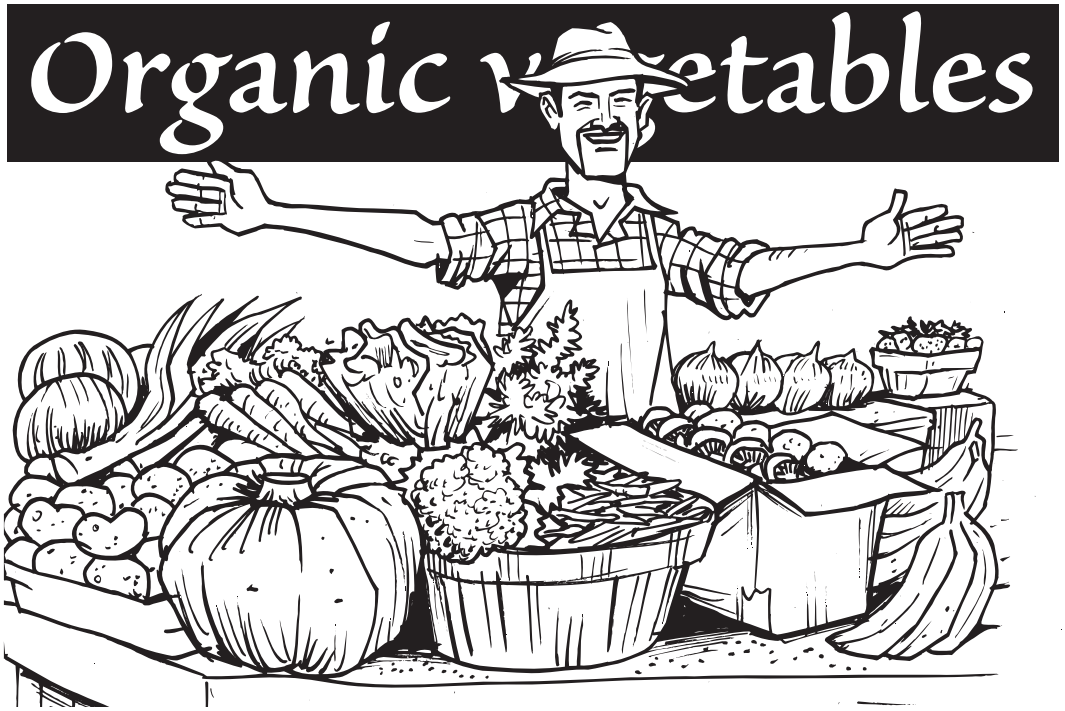
He brought 66 pumpkins to the market.

Possible difficulties
- Unable to see how the half tomato or pumpkin or extra tomatoes or pumpkins fit into the problems
- Simply working on the basis of calculations with the numbers in the problem to obtain incorrect answers

Extension
- Change the numbers in the problems but leave the problem statements the same:
  – More avocados for sale, a different amount more or less than 1 half, 1 third, 3 quarters, …
  – More avocados or pumpkins at the end of the sale or 3, 4 or 5 more each time.
- Have pupils make up problems like these of their own and challenge others to solve them using diagrams, tables for try and adjust or algebraic thinking.

SAMPLE - NOT FOR SALE

THE FARMERS MARKET



1. Lance grows and sells organic vegetables at the local farmers market. He sold 572 avocados at the two markets held last weekend. When he took stock on Sunday night, he noticed that the number of avocados he sold on Sunday was 25 less than half the number he sold at the Saturday market. How many avocados did he sell at each market?

2. Lance is really well known for his luscious tomatoes. On Saturday, he sold half of his tomatoes and another half a tomato to his first customer, half the remaining tomatoes and another half a tomato to his second customer, half the remainder and another half a tomato to his third customer. He then sold the remaining 55 tomatoes to the woman who ran the sandwich stall. How many tomatoes did he have to start with?

3. As the weather had turned cold, people wanted to make soup and there was a rush on pumpkins as soon as Lance opened his market stall. The first customer bought half the pumpkins he had brought to the market and three more pumpkins. His next customer bought half of those pumpkins and another three pumpkins. The third customer bought half of what he had left and another three pumpkins. The fourth customer quickly bought the last three pumpkins. How many pumpkins did he bring to the market to sell? (Lance only sold whole pumpkins as he found it too hard to cut them in half—he always got two unequal parts.)

SAMPLE - NOT FOR SALE

Problem-solving
To read, interpret and analyse information

Curriculum links
Sixth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Record the processes and results of work using a variety of methods
- Represent mathematical ideas and processes in different modes
- Reason systematically in a mathematical context
- Use mental strategies and procedures for carrying out mathematical tasks

Number
- Explore square numbers

Materials
calculator

Focus
These pages explore concepts of place value, number sense and using data. The relationships among numbers and place value is analysed and pupils are encouraged to not only find possibilities but to also disregard numbers and combinations that are not possible. Data needs to be interpreted and analysed to find solutions. See the note on using a calculator (page xx) for ways to use the different functions.

Discussion
Page 9
This investigation requires pupils to read and interpret information and use this information to find a solution. Pupils need to think in terms of fixed costs occurring regardless of income and variable costs occurring according to production. The variable cost per item is not given as such and pupils need to use information regarding 500 items which have a variable cost of €7500 to identify that the variable cost per item is €15.

Once the table is complete the data can then be used to work out at which point the factory moves from making a loss to making a profit. The point at which this occurs is not shown in the table and pupils need to take what they know—that she is making a loss at 100 items but making a profit at 250 and somewhere in between is where the break-even point will occur.

Page 10
These problems are easy to state and investigate using a calculator's memory function to simplify the steps and assist in developing algebraic thinking as the patterns described in words for the examples used are translated into early proofs in general. The first problem investigates a pattern based on the difference of two squares that was first used in Book E, but now the focus is on why the result would be true in general. The second problem is an investigation based on the difference of two cubes. The pattern can be discerned as the examples are worked through using the calculator, but showing why this is true in general requires quite sophisticated algebraic reasoning which should be understandable by the children but not necessarily a form of thinking they would come to by themselves.

Page 11
The puzzle scrolls contain a number of different problems, all involving strategic thinking to find possible solutions. In most cases pupils will find tables, lists and diagrams are needed to manage the data while exploring the different possibilities. In some cases the information needs to be analysed to find a pattern to find out solutions. For example, in the investigation involving '3 digit numbers adding to 6' the possible solutions from numbers in the one hundreds can then be used to determine other numbers in the 2 hundreds and so on.

The scroll exploring 3-digit numbers when dividing by 5 and 7 would mean that each number had to end in a 3 or an 8. The first 3-digit number divisible by 7 is 105. If we add 2 to this number we get 107 which can be our starting number for looking for other numbers. If we key 107 into a calculator, add 7 and press equal it will keep adding 7 each time. If we then record any numbers which end in a 3 or an 8 and we have the numbers which will result in a remainder of 3 when dividing by 5 and a remainder of 2 when dividing by 7.

For the scroll exploring the number of beads pupils can again think in terms of multiples. If they find 13 by 8 (the first 3-digit multiple of 8) which is 104, add one to this number to give the starting number. Again a calculator can be used (add 8, press the equal key and it will add 8 each time). These numbers can be recorded and pupils can then look at multiples of 7 (with 1 added) and highlight any numbers that are the same and then, from these possibilities, check to see if it is also true for 6.

Possible difficulties
- Difficulty with the concepts of fixed cost and variable costs
- Difficulty with the concept of profit and loss
- Poor understanding of place value
- Wanting to add, subtract or multiply rather than using place value or number sense
- Not using all the criteria

Extension
- Pupils could think up their own profit and loss problems using different criteria.
- Pupils could make puzzle scroll cards where they take the existing card and change the numbers or context to make a new problem.

SAMPLE - NOT FOR SALE

PROFIT AND LOSS

A factory owner knows that some of her expenses are going to occur regardless of how many items she makes and sells. These are fixed costs and include things such as rent, insurance, phone and Internet. Her fixed costs total €4000 per week.

Her other expenses are variable costs which change each week depending on the number of items she makes. Variable costs include things such as materials, labour and electricity.

All factory items are made to order, so all items produced are sold. In one week she sold 500 items and her variable costs were €7500. All items sell at €50 each.

1. Complete the table below to show the income for the shop based on the number of items sold as well as the total costs for each week.

Items sold per week	50	75	100	250	500	1000	1500	3000
Income								
Total costs								

The difference between total costs and income is the factory owner's profit. If she made no sales she would still have to pay the fixed costs and would have a loss for that week.

2. Did she make a profit or a loss when she sold 100 items?

_____

3. Did she make a profit or a loss when she sold 1000 items?

_____

4. Using the information in the table, estimate the number of items she needs to sell each week to break even. Calculate the expenses and income for that number of items to check your estimate.

_____

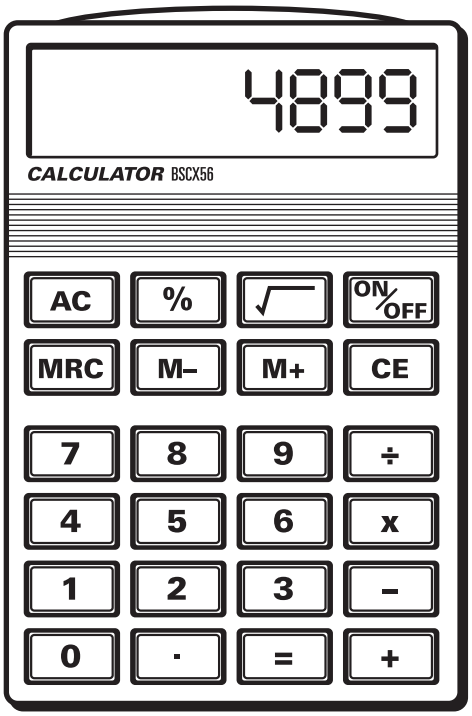
_____

SAMPLE - NOT FOR SALE

CALCULATOR PATTERNS

Use your calculator to help think about what is happening.

Squaring and multiplying



1. Choose 3 consecutive 2-digit numbers; e.g. 69, 70, 71.
   (a) Multiply the first and third numbers.
   (b) Square the middle number.
   (c) What do you notice?

   _____

   _____

   (d) Try some other 2-digit numbers.
   (e) Try some 3-digit numbers and 4-digit numbers.
   (f) Describe the pattern. Why does this happen?

   _____

   _____

Cubing, squaring and multiplying

2. Choose a 2-digit number; e.g. 47.
   (a) Make a new number by finding the difference between the cube of the tens digit and the cube of the ones digit.
   (b) Make another new number by adding the square of the tens digit, the square of the ones digit and the product of the tens and ones digits.
   (c) Divide the first new number by the second new number.
   (d) Try some other 2-digit numbers.
   (e) What do you notice?

   _____

   _____

   (f) Investigate what happens if you split a 3-digit number into 2 parts (there will be two different ways).

   _____

   _____

   (g) Describe the pattern. Can you explain why this happens?

   _____

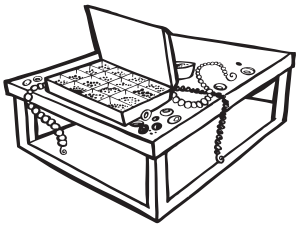
   _____

SAMPLE - NOT FOR SALE

PUZZLE SCROLLS

1. Five paintbrushes and 2 sets of paints cost €19.25 while 3 brushes and 2 sets of paints cost €16.75. How much does 1 set of paints cost?

2. A box with over 100 beads can be shared among 6, 7 or 8 bags with 1 bead left over each time. What is the smallest number of beads that the box could have?



3. The number 105 has a digit sum of 6 (1 + 0 + 5). How many other 3-digit numbers have a digit sum of 6?

4. When 54 is added to a number the result is the same as when the number is multiplied by 3. What is the number?

5. What 3-digit numbers have a remainder of 3 when divided by 5 and a remainder of 2 when divided by 7? How many did you find?

6. Two cash registers had a combined total of €500. When €18 was taken from one register to the other, they each had the same amount. How much did they have before the transfer?

SAMPLE - NOT FOR SALE