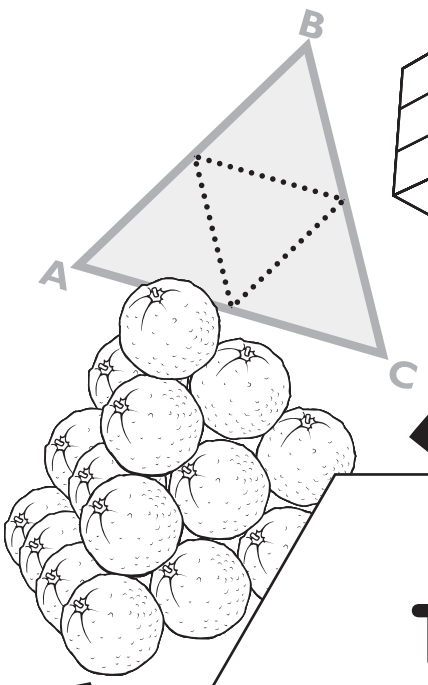
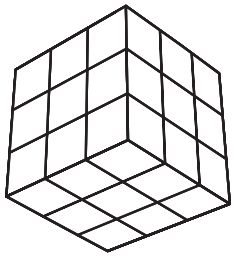
PRIMARY

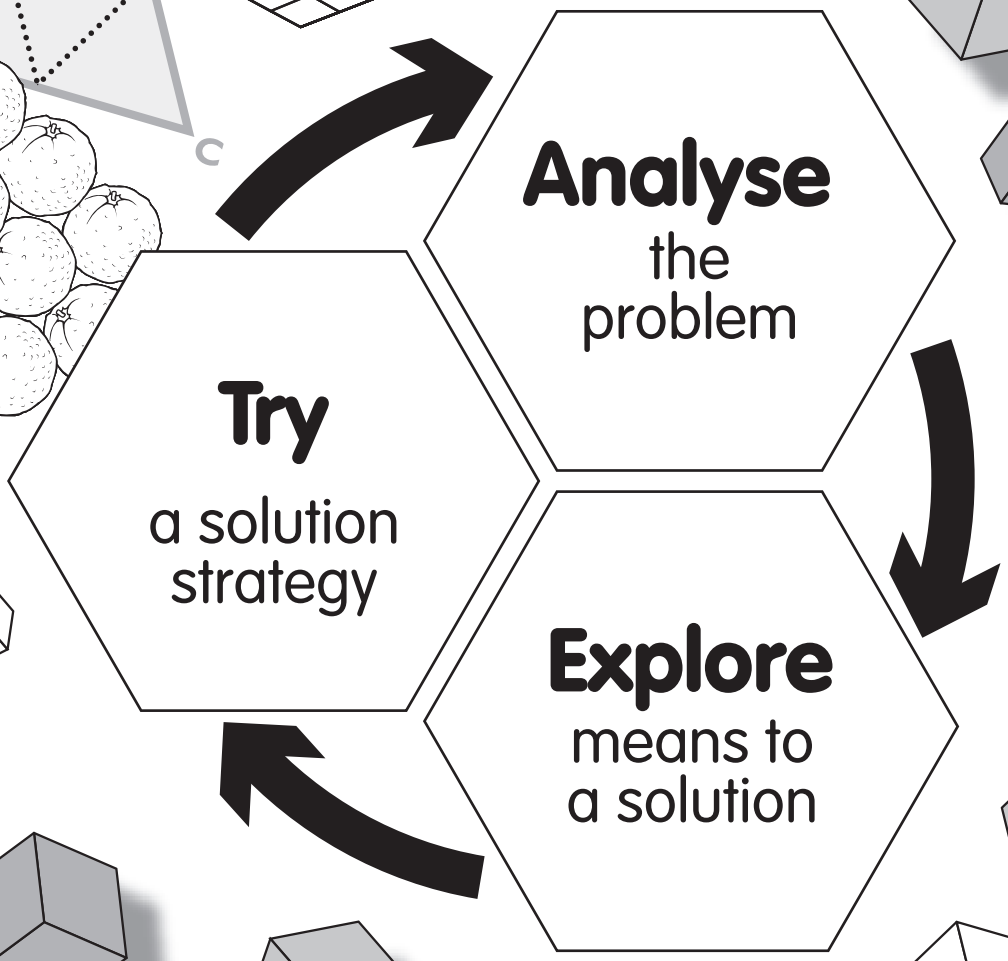
BOOK D
(Ages 8–9)

CURRICULUM LINKED

Problem-solving in mathematics







George Booker and Denise Bond

SAMPLE - NOT FOR SALE

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Problem-solving
To use spatial visualisation and logical reasoning to solve problems.

Curriculum links
Third Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Represent mathematical ideas and processes in different modes: pictorial and diagrammatic
- Explore and investigate mathematical patterns and relationships
- Use appropriate manipulatives to carry out mathematical procedures

Shape and space
- Solve and complete practical tasks and problems involving 3-D shapes

Materials:
Cubes such as wooden, Multilink™ or Unifix™ cubes.

Focus
These pages explore arrangements and dissections of three-dimensional shapes in order to determine how particular outcomes are formed. Spatial as well as logical thinking and organisation are involved as pupils investigate all likely arrangements to ensure that the final forms match the given criteria or visualise a given shape in terms of its component parts.

Discussion
Page 3
Pupils need to be able to visualise the arrangements of cubes, oranges and cans stacked in several layers.

Some will see the first drawing as consisting of one layer of 16 cubes, another of nine cubes, another of four cubes and then one cube on top. Others will see slices going down the staircase—10, then 9, 7 and 4—or see these building upwards.

They will need 30 cubes to build the arrangement. The different ways that the diagram can be visualised and the staircase built should be discussed with the class.

The stacks of oranges and cans use either a triangular base or square base and have the shapes stacked on the intersection of the ones below rather than one on top of another.

The final shape does not follow a simple pattern and pupils will need to visualise how the blocks are stacked and find a way to organise their count. This will influence the building of their own structures and how they visualise and keep track of the individual cubes.

Page 4
Seeing how cubes make a larger structure is now extended to include applying paint to a completed shape and visualising the effect on the individual cubes. Pupils may need to build the structures and manipulate blocks in order to see what happens. Some pupils may need to place labels on certain faces of the cube to help them come to terms with what the questions are asking.

Encourage the systematic analysis of the shapes when discussing the results: Could any cubes have all sides painted? Where would cubes with six, four, three, two or one side(s) painted be found? Which of these are possible and which are not possible on each shape?

Page 5
The problems on this page extend the thinking needed to visualise and see patterns. When the large cube was cut into smaller cubes, some of the smaller cubes would have been hidden inside the whole and careful analysis is needed to determine the way individual cubes would be painted. A table of possibilities is one way of keeping track of what happened.

Possible difficulties
- Unable to visualise the cubes in the representations on the two-dimensional page
- Only considers the cubes that can be readily seen on the outside of the shapes

Extension
- Have pupils make stacks using different arrangements of cubes and others work out how many were used.
- Use isometric paper to draw the stacks they make and have other pupils see how many cubes are used.
- Extend the problems by asking what would happen if more blocks had been used—if each arm of the 'T' had one, two or more additional cubes before painting. What if the large block had been cut into cubes with sides of one centimetre? What if the cube was cut into 64 or 125 smaller cubes?

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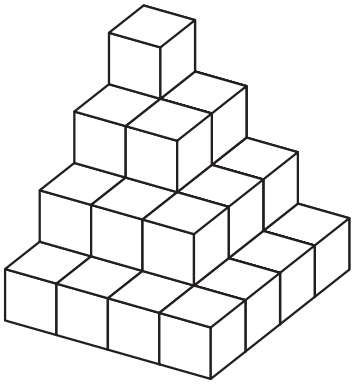
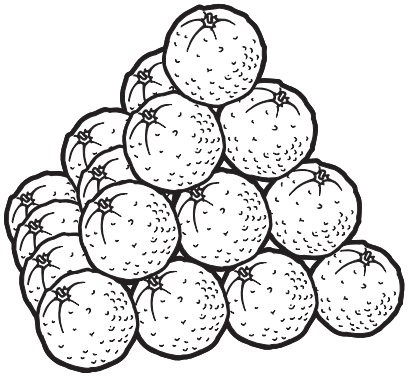
STACKING SHAPES

1. A set of steps was made by stacking one cube on top of another. How many cubes were needed to make the staircase?

_____



Find as many cubes as you think you need and make the staircase.

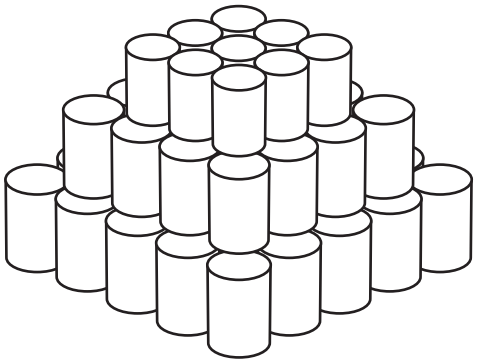


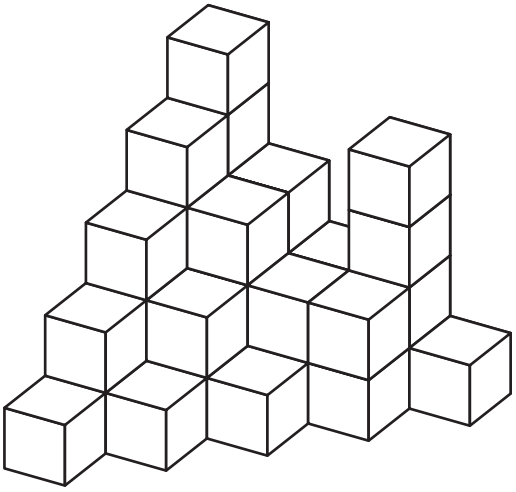
2. Greengrocers often stack fruit as a pyramid. How many oranges are in this stack?

_____

3. At the supermarket, you sometimes see cans of soup stacked for a display. How many cans are in this stack?

_____





4. These cubes have been stacked one on top of another. Some of the cubes are hidden behind or beneath others and cannot be seen. How many cubes were used to build the shape?

_____

Get some cubes and build your own stack. Write the number of cubes you used and challenge a friend to work out how many you used without pulling your stack apart.

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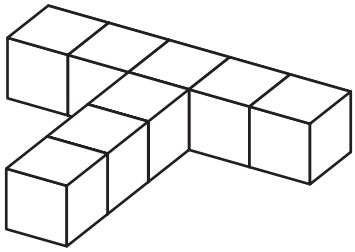
PAINTED CUBES



This shape was made using eight cubes. After they were joined together, the final shape was painted green on all of its sides. When it was taken apart again, some of the faces of the cubes have green paint and others do not.

1. How many of the individual cubes would have green paint on exactly four faces?

   _____

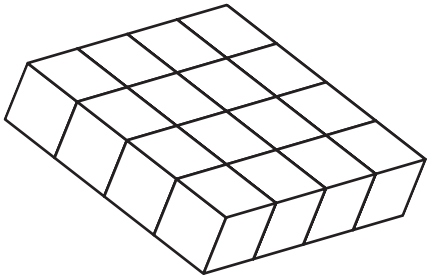
2. Would any of the individual cubes have five faces painted? How many?

   _____

3. Would any have three faces painted? How many?

   _____

A wooden block is 8 cm long, 8 cm wide and 2 cm high. The block is painted blue on all six faces and then cut into 16 cubes, each with sides of 2 cm.



4. How many of the cubes would have blue paint on four faces?

   _____

5. How many of the cubes would have only three faces painted?

   _____

6. How many of the cubes would have only two faces painted?

   _____

7. Why would there be no cubes with only one, five or six faces painted?

   _____

   _____

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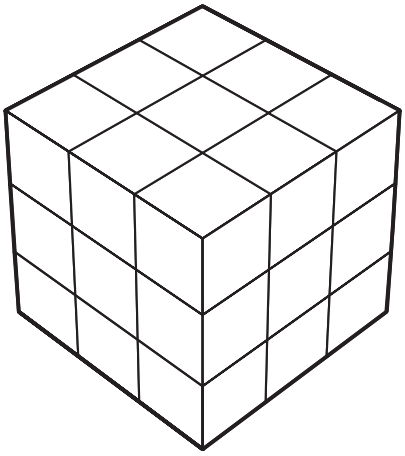
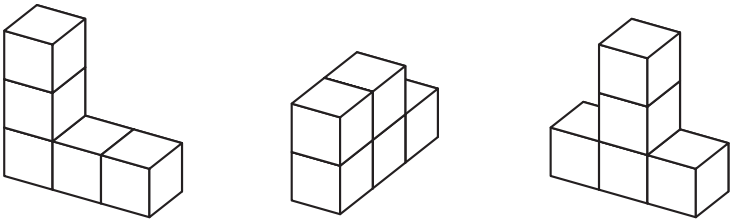
CUBE PAINTING

In the art gallery, a large polystyrene cube was hung from a string and spray-painted red on all six faces. It was then cut into twenty-seven smaller cubes to be used by children to make shapes of their own.





1. With the large cube cut, how many of the individual cubes would have red paint on only three faces?

   _____

2. Would any of the cubes have no red paint? _____
3. How many? _____
4. Describe the cubes.

Number of faces painted	Number of cubes
0	
1	
2	
3	
	Total =

5. When you count all of the cubes you have described, are all 27 included?

   _____

SAMPLE - NOT FOR SALE

Problem-solving
To use strategic thinking to solve problems.

Curriculum links
Third Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Explore and investigate mathematical patterns and relationships

Number
- Read and write three-digit numbers

Algebra
- Explore, recognise and record patterns in number

Materials
0–99 number board

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Focus
This page explores pupils' understanding of the number system and their ability to solve questions about numbers. Pupils need to coordinate the reading and writing of numerals with the symbols involved in writing the numbers 400–600.

Discussion
Page 7
This problem explores pupils' ability to reason about the number system and keep track of the possibilities they find. Pupils need to discuss what it means to 'say' a number as opposed to 'writing' a digit.

When pronunciating certain numbers—for example, with numbers which include 'four'—it is important for pupils to include numbers which include the pronunciation of the word part 'four' within 'fourteen', 'forty', 'forty-one' etc.

If pupils take the problem further and try other number ranges, as suggested, they will find a different pattern altogether for the three hundreds as opposed to the four hundreds. After the five hundreds the patterns begin to repeat. When pupils notice this, they will have really come to terms with the strategic thinking needed to organise and solve problems with several interacting conditions.

Trying different number ranges or counting and writing in fives and 25s will provide different patterns as pupils coordinate what happens to the ones, tens and hundreds digits.

Possible difficulties
- Unable to keep track of the number of times they determine a digit or word
- Confusion between saying and writing the digits
- Not understanding that we say 'four' in numbers which contain 'forty', even though it is written differently to 'four' or 'fourteen'
- Thinking that 'five' is said for numbers with 'fifty'
- Confusion with 44, 55, etc. including only seeing the digit once when it actually occurs in both the ones and tens place

Extension
- Try other number ranges such as 300–500, 400–700 etc.
- Investigate the number of times 'five' is said and written when counting by fives or 25s.
- Make a table to display the results and present a description of the problems and their solution to another class.

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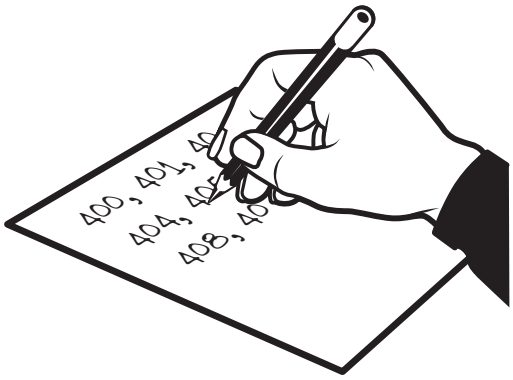
HOW MANY DIGITS?

1. (a) How many times do you say 'four' when you count from four hundred to six hundred?

_____



(b) How many times do you write '4' when you write all of the numbers from 400 to 600?

_____

2. (a) If counting from four hundred to six hundred, would you say 'five' more or fewer times than you would say 'four'?

_____

(b) How many times would you say 'five'?

_____

(c) How many times would your write '5' when writing all of the numbers from 400 to 600?

_____

3. Try other 1-digit numbers. Can you see a pattern?

_____

_____

SAMPLE - NOT FOR SALE

Problem-solving
To identify and use concepts about numeration.

Curriculum links
Third Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Explore and investigate mathematical patterns and relationships

Algebra
- Explore, recognise and record patterns in number

Materials
counters, blocks or a calculator

Focus
These pages explore solving problems involving number sense, magic squares and logic. Pupils need to carefully analyse the problems to locate information necessary to find the magic number or the arrangement of numbers. Counters, blocks or a calculator can be used to assist as these problems focus on the concepts of number sense and number logic rather than basic facts.

Discussion
Page 9
This activity investigates possible combinations of numbers to make a total. Pupils have four scores which add together to make a total score and a list of numbers which make up the scores. Number 'sense' and the 'try and adjust' strategy are needed to find solutions.

For example, the score of 32, using the digits 5 and 9, requires the combination of 9, 9, 9 and 5; while a score of 57, using 24, 7 and 13 requires the combination of 24, 7, 13, 13.

Page 10
This investigation involves the concept of magic squares. Simple 3-by-3 magic squares have been used to establish the idea that each row, column and diagonal adds to the same magic number. In this case, the concept is further explored as pupils investigate a famous magic square.

The Dürer magic square has a number of points of interest aside from being a magic square. For example, not only does each row, column or diagonal add to the same number, so do the four corners, the four middle numbers and the four corner mini (2-by-2) grids. Also, it uses the numbers 1 to 16 and shows the year in which it was constructed.

Page 11
This page introduces pupils to the concept of sudoku. The word 'sudoku' roughly means 'digits must only occur once'. In this case a 4-by-4 grid has been used and every row, column and mini-grid must contain one of each of the numbers 1, 2, 3 and 4. More commonly a 9-by-9 grid is used, using the digits 1 to 9. No addition or basic facts are involved and pupils need to use logical reasoning to find solutions.

Possible difficulty
- Considering only rows or columns rather than rows, columns and diagonals in the magic squares or the smaller grids in the sudoku puzzles

Extension
- Investigate other magic squares and magic numbers.
- Explore sudoku games in magazines, newspapers and on the Internet that involve 4-by-4 grids as well as 9-by-9 grids.

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STAR GAZE

Star gaze is a new computer game.
Each time you win, you collect stars.
Total the four stars to find your score.

1. Work out these scores.

(a) 5 9 6 3

Score: _____

(b) 17 19 15 4

Score: _____

(c) 15 33 24 33

Score: _____

2. Add combinations of the numbers given to find the score.

(a) **Score: 27**
use only 3 and 8

(b) **Score: 32**
use only 5 and 9

(c) **Score: 26**
use only 6 and 7

(d) **Score: 16**
use only 5, 2, 7

(e) **Score: 21**
use only 8, 1, 4

(f) **Score: 28**
use only 9, 5, 7

(g) **Score: 57**
use only 24, 7, 13

(h) **Score: 71**
use only 6, 19, 23

(i) **Score: 99**
use only 23, 37, 16

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MAGIC SQUARES

In magic squares, all of the numbers in each row, column and diagonal add to the same total.

7	0	5
2	4	6
3	8	1

1. This magic square has a magic number of _____.

Complete the magic squares below. Remember, all rows, columns and diagonals must add to the same total.

2. (a)

		11
	9	
7		3

Magic number: _____

(b) What do you notice about each number in the square?

_____

3.

11		9
	8	
		5

Magic number: _____

FAMOUS MAGIC SQUARES

16			13
5	10		8
9		7	
	15		1

The following magic square was constructed in 1514 by an artist named Albrecht Dürer. Complete the magic square and discover why it is so famous.

4. Magic number: _____
5. What do the four corners add to? _____
6. What do the four middle squares add to?

   _____
7. What do the four squares in each corner add to?

   _____
8. What digits have been used? _____
9. Can you see anything to do with the date it was first constructed?

   _____

SAMPLE - NOT FOR SALE

SUDOKU

Sudoku puzzles are made up of numbers and to solve them you must use logic to work out where the numbers go.

Every row, column and mini-grid must contain one of each of the numbers 1, 2, 3 and 4.

2	1	3	4

Row – has the numbers 1, 2, 3 and 4.

			4
			1
			2
			3

Column – has the numbers 1, 2, 3 and 4.

		3	4
		2	1

Mini-grid – has the numbers 1, 2, 3 and 4.

2	1	3	4
3	4	2	1
4	3	1	2
1	2	4	3

The completed sudoku has the numbers 1, 2, 3 and 4 in every row, column and mini-grid.

Complete each sudoku.

1.

1	2		3
	4	1	
	1	3	
4		2	1

2.

	1		2
		1	3
1	3		
4		3	

3.

	1	2	
	2	3	
	3	4	
	4	1	

4.

	1	2	
3		1	
	4		2
	3	4	

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