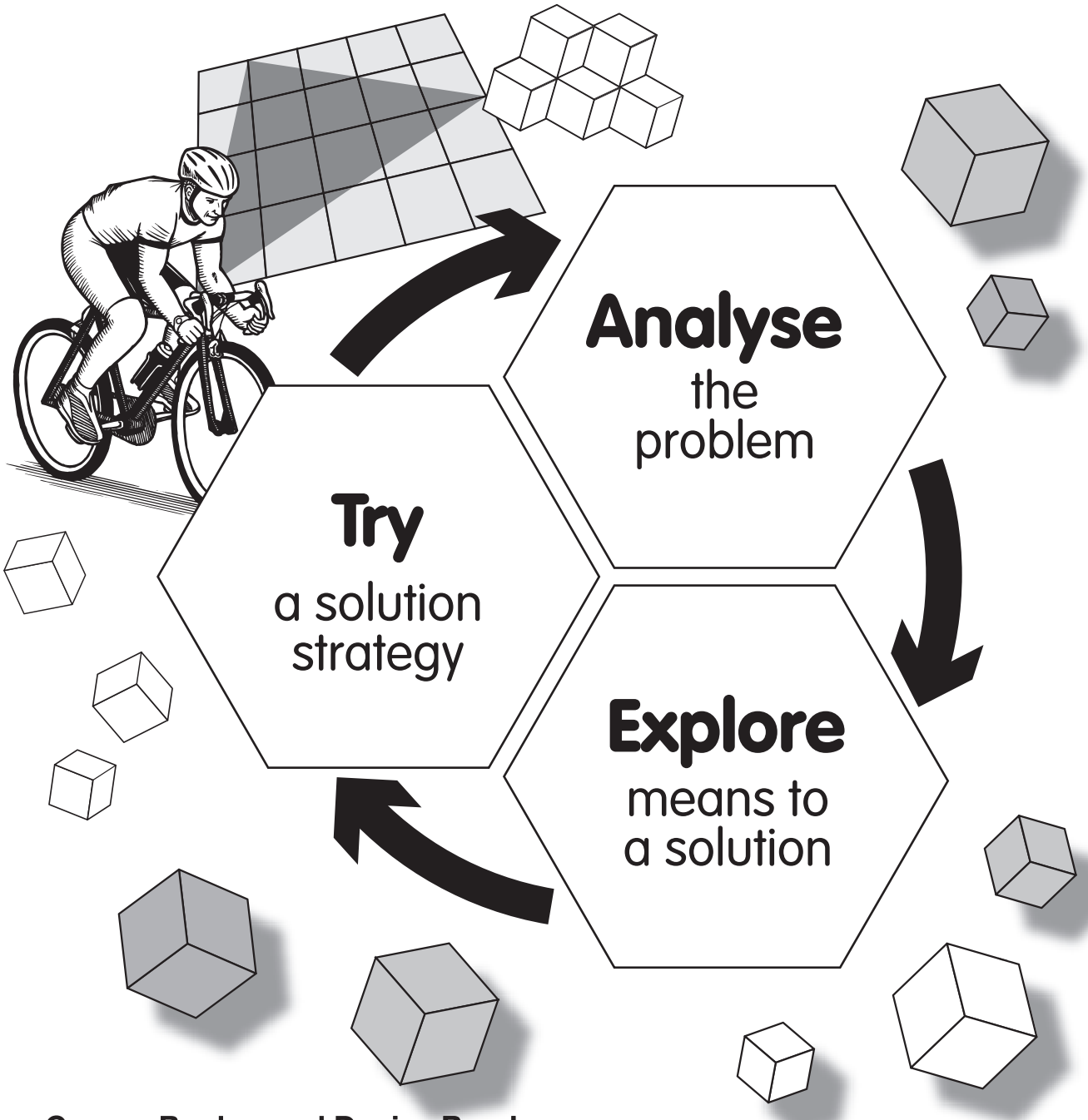
PRIMARY

BOOK F
(Ages 10–11)

CURRICULUM LINKED

Problem-solving in mathematics



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SAMPLE - NOT FOR SALE

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Problem-solving
To use spatial visualisation and logical reasoning to solve problems

Curriculum links
Fifth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Record the processes and results of work using a variety of methods
- Represent mathematical ideas and processes in different modes
- Search for and investigate mathematical patterns and relationships

Number
- Identify multiples

Materials
cubes in 4 different colours, grid paper to draw views of the shapes and nets of cubes, digital camera to photograph the shapes

Focus
These pages explore arrangements of three-dimensional shapes in order to determine how particular outcomes are formed and to investigate the patterns they exhibit. Spatial and logical thinking and organisation are required as pupils investigate all likely arrangements to ensure the final forms match the given criteria or visualise a given shape in terms of its component parts.

Discussion
Page 3
This problem requires pupils to visualise how coloured cubes are arranged to form larger cubes. Pupils need to bear in mind that a cube with a height and width of 2 cubes will have a total of 8 cubes, a 3-by-3 cube will have 27 cubes, and so on—multiplying side x side x side (or side^3) to find the total. Pupils can then determine the difference in amount of cubes used; e.g. from a single cube to a 2-by-2 cube: 2 x 2 x 2 (or 2^3) – 1 = 7 cubes. The next difference is 27 – 8 = 19, and so on. Other pupils will see this pattern in terms of the amounts of cubes that are added to build the next cube. Both approaches are satisfactory and will assist in helping pupils see a pattern in terms of how many cubes each subsequent amount increases by; they increase by sequential multiples of six.

Page 4
Pupils are asked to extend an arrangements of cubes in a Latin square (where each row and column has exactly one of each of 3 or more different coloured cubes), to a cube in which each horizontal and vertical row or column has exactly one cube of each colour. Pupils will need to visualise these patterns to guide their building of the complete cubes and some degree of 'try and adjust' will probably be needed. Encourage pupils to create systematic arrangements as they complete the tasks and describe the pattern of the blocks in the middle of the arrangements. This may lead some pupils to start with the 3-by-3 cube and build layers all around to complete a 4-by-4 cube. All arrangements would be possible with a single cube at the centre of an arrangement of an odd number of colours, while arrangements made of an even amount of colours would have a 2-by-2 cube in their centre. The actual descriptions given will vary according to the arrangement chosen for the initial Latin square, but all are essentially the same with different colours occupying different positions.

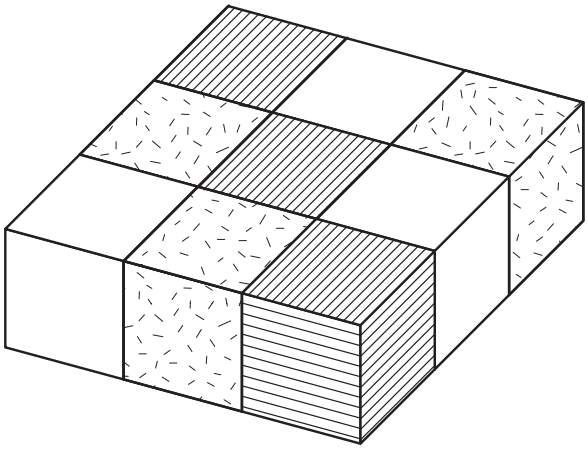
Page 5
This set of investigations encourages pupils to see three-dimensional shapes in terms of the component two-dimensional forms. Viewing from above, the front and one side is sufficient to capture the details of a three-dimensional shape. Another way of providing a plan for a building is to use a grid that tracks the number of cubes in each position of the base. After they make a shape of their own and construct a grid plan for another pupil to follow, they may be able to take digital photos from the side and front to compare with the original.

Possible difficulties
- Unable to visualise the shapes from the two-dimensional representations
- Unable to keep track of the possibilities in any of the investigations

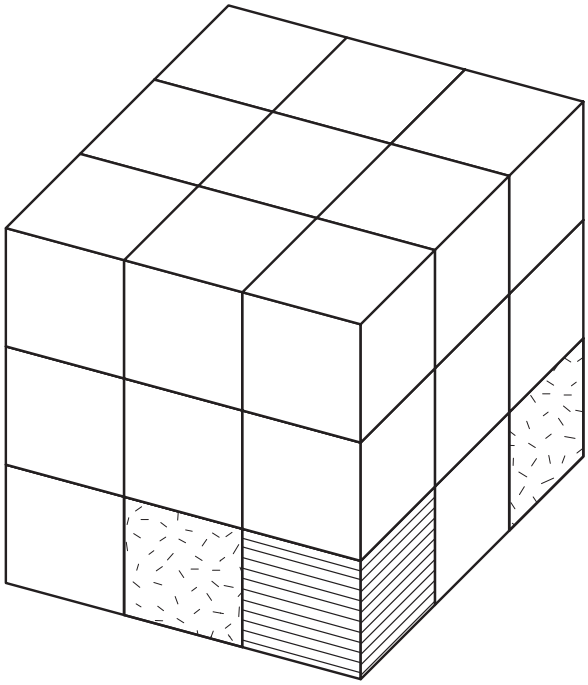
Extension
- Investigate the pattern formed by the number of small cubes faces on the outside of the growing cubes: 6, 54 …
- Build a shape from cubes then photograph or draw the front, side and top views. Ask another pupil to draw a grid plan from the pictures. Make the shape from the plan and compare with the original.

SAMPLE - NOT FOR SALE

COLOURED CUBES



Use cubes in three different colours to make a square in which each row and column has exactly one cube of each colour.



Can you build this to be a cube so that on every face each row and column has exactly one cube of each colour?

1. How many cubes of each colour did you use? _____

2. (a) What coloured cube would be at the centre of your larger cube? _____

   (b) Can it be any other colour? _____

See if you can make an even larger cube from cubes in four different colours and still keep the same pattern.

3. How many of each coloured cube do you need? _____

4. What would the colour of the cube be at the centre of the larger cube? _____

   _____

5. Is it possible to continue forming larger cubes in this way? _____

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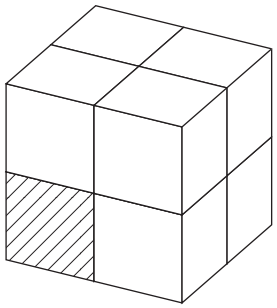
GROWING CUBES

Place a single cube in front of you. Imagine you add cubes of a second colour to it to make a larger cube.



1. How many extra cubes would you need? _____

Now imagine that you add cubes of a third colour to make a larger cube again.

2. How many cubes would you need to make a bigger cube? _____

3. How many cubes of a 4th colour would you need to add to make an even larger cube?_____

Get as many of the coloured cubes as you think you need to increase the size. See if you can make the larger cube as you imagined.

4. Explain to a friend how you worked out the number of cubes you needed to increase the size. Did you both do this the same way?

_____

_____

_____

_____

_____

5. List the number of cubes that you would need to add if you kept growing cubes in this way.

1 + 7 + _____ + _____ + _____ + _____ + _____

6. Can you describe a pattern for the way the cubes increase in size?

_____

_____

_____

_____

_____

_____

_____

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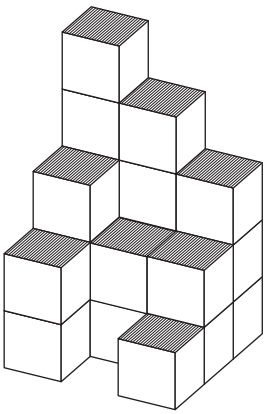
VIEWING CUBES

1. Draw what this shape would look like from the:



(a) front. (b) side.

Use some cubes to make the shape.

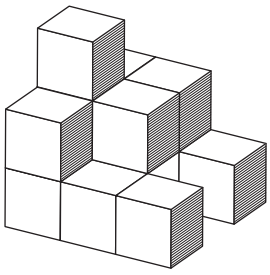
Here is a plan of this shape, using squares to show where the cubes are and numbers to show how many cubes are used.

5	4	3
3	2	2
2		1

2. Does it match the shape you made? _____

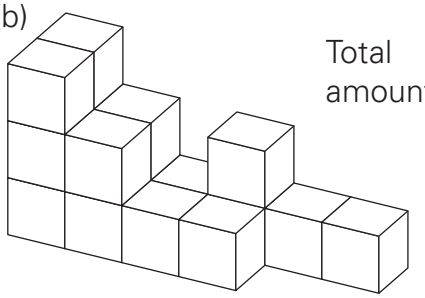
3. Write the number of cubes you would need to make each of these shapes. Get the cubes, make the shapes and draw a plan for each one.

(a)



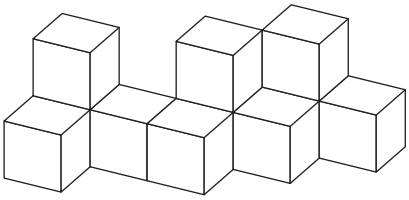
Total amount = _____

(b)



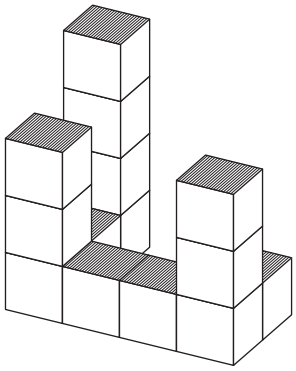
Total amount = _____

(c)



Total amount = _____

(d)



Total amount = _____

4. Make a shape of your own and then make a plan for it. Ask a friend to construct the shape from your plan and compare.

SAMPLE - NOT FOR SALE

Problem-solving

To use logical reasoning and number sense to solve problems

Curriculum links

Fifth Class

Skills

- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Record the processes and results of work using a variety of methods
- Represent mathematical ideas and processes in different modes
- Devise and use mental strategies and procedures for carrying out mathematical tasks
- Use appropriate manipulatives to carry out mathematical procedures

Number

- Solve problems involving operations with whole numbers and fractions

Materials

counters, Base 10 materials

Focus

This page explores problems based on a conceptual understanding of whole numbers and fractions and an understanding of logical reasoning to take into account the problem's context. Backtracking from the final position will assist in understanding and solving the problems but counters could also be used to keep track of what is happening. Using a diagram or calculator are other ways to sort through the information while keeping the intent of the problem in mind.

Discussion

Page 7

These problems highlight the need to carefully analyse the problem before starting on a solution. For the first problem, Lindsay is clearly not actually going to sell half an egg! At each step, there must be an odd number of eggs. When she sells these eggs, they will both get half of the nearest even number and the customer will get 1 extra egg. This means that when Lindsay had 9 eggs, the last customer must have taken 10. Logic dictates that Lindsay would have had 19 eggs to sell to her third customer. The customer before that must have taken 20 eggs when Lindsay had 39 eggs to sell. So the first customer took 40 eggs when Lindsay had 79 eggs to start with. Counters can be used to work through this thinking and a diagram can help keep track of what is needed:

Lindsay	third customer	Lindsay	second customer	Lindsay	first customer	Lindsay
9	10					
		19	20			
				39	40	
						79

A diagram can also be used for the second problem to determine the relationships among the information. Base 10 materials could also be used to model the 170 eggs and how they need to be distributed over the markets so that, at the second market, the number is 10 less than half of the first.

The third problem is similar to the first problem but this time the seller must add 2 to the number of chickens left to find half of the number he had to sell. Counters or a diagram could readily show this.

The last problem can also be solved using counters or numbers in a table. At first it seems that there must be 4 or 8 children. When these prove impossible, other ways of finding whole numbers of loaves need to be considered while bearing in mind that the number of people and the number of loaves must both equal 12.

Possible difficulties

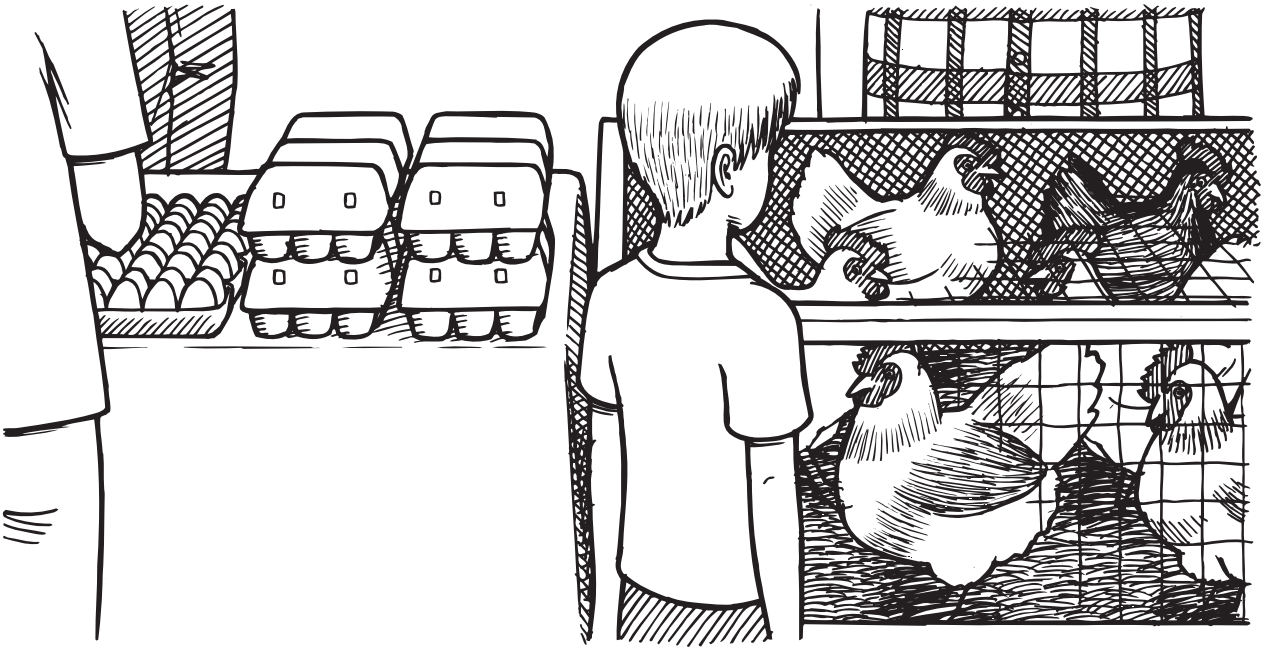
- Unable to see how the half egg, 10 more eggs or 2 extra chickens fit into the problems
- Not thinking that a full loaf could be made up of 2 halves, 1 half and 2 quarters or 4 quarters to see how the 12 full loaves could be bought
- Simply working on the basis of calculating the numbers in the problem to obtain incorrect answers

Extension

- Ask for the number of eggs or chickens each customer bought.
- Change the numbers in the problems but leave the problem statements the same:
  - more eggs at the end of the first problem (it must be an odd number)
  - less eggs at the second market (55, 85, 115, … eggs altogether)
  - more chickens available at the end of the day or 3, 4 or 5 more each time
  - the fourth problem is a very famous problem and would be difficult to change!
- Change the problem's context but leave the numbers the same.
- Have pupils make up problems like these of their own and challenge others to solve them using diagrams or materials.

SAMPLE - NOT FOR SALE

MARKET DAYS



1. My cousin, Lindsay, keeps chickens at her home and sells any spare eggs at the local farmers' market. She sold half of her eggs and another half an egg to her first customer, half the remaining eggs and another half an egg to her second customer and half the remainder and another half an egg to her third customer. At the end of the day, Lindsay noticed that she had 9 eggs left. How many eggs did she have to start with?

   _____

2. Lindsay sold 170 eggs at two different markets. She noticed that the number she sold at the second market was 10 less than half the number she sold at the first market. How many eggs did she sell at each market?

   _____

3. There was a rush to buy fresh chickens when the chicken farmer opened his market stall. The first customer bought half of the chickens he had brought to the market and two more chickens. His next customer bought half of the remaining chickens and another two chickens. The third customer bought half of what he had left and another two chickens. Sadly, the fourth customer was only able to buy one chicken. How many chickens did the chicken farmer bring to the market to sell?

   _____

4. After the market, Lindsay joined a group of people going on a picnic. The twelve of them went first to the bakery and bought a dozen loaves of bread. Each man bought two loaves, each woman bought half a loaf, and each child a quarter of a loaf. How many men, women and children were at the picnic?

   _____

SAMPLE - NOT FOR SALE

Problem-solving
To read, interpret and analyse information

Curriculum links
Fifth Class
Skills
- Apply concepts and processes in a variety of contexts
- Analyse problems and plan an approach to solving them
- Select and apply a variety of strategies to complete tasks and solve problems
- Record the processes and results of work using a variety of methods
- Represent mathematical ideas and processes in different modes
- Devise and use mental strategies and procedures for carrying out mathematical tasks
- Use appropriate manipulatives to carry out mathematical procedures

Number
- Identify place value in whole numbers

Materials
calculator, number expanders

Focus
These pages explore concepts of place value and number sense. The relationships among numbers and place value is analysed and pupils are encouraged to not only find possibilities but to also disregard numbers and combinations that are not possible. Place value and number sense are needed, rather than addition or multiplication.

Discussion
Page 9
This investigation requires pupils to read and interpret information and use this information to find a solution. As the book has 18 chapters of equal length, a calculator can be used to easily work out how many pages are in each chapter. This information can be then readily used to calculate the last page of each chapter and subsequently the starting page of the following chapter. Before starting pupils may find it helpful to draw up a table outlining the starting page, the halfway point and the last page of each chapter. We know that each chapter has 124 pages and that Chapter 1 starts on page 1, so Chapter 2 must start on page 125 and so on. Entering 124 + 124 and hitting the '=' key on the calculator over and over will give you the last page of each of the 18 chapters. The page number that Nathan has read to is already provided and this information is needed to answer some of the problems; for example, in order to determine about how many pages Nathan has read past the middle of the book it is necessary to keep in mind that there are 2232 pages in the book and that he has already read up to page 1674. Similarly, this information is needed to determine how many pages need to be read to finish the book.

Page 10
These problems are very easy to investigate and solve by using a calculator, but require a deep understanding of place value to work out ***why*** they happen. The first problem sets out a pattern of investigation for forming two-digit numbers from any three different digits and dividing the sum of the numbers formed by the sum of the digits. The result is surprising at first, but the reason is found by considering how the digits are placed in each number. The problems then continue to include other digits, reinforcing the need to look for patterns and to understand place value.

Page 11
The puzzle scrolls contain a number of different problems, all requiring strategic thinking to find possible solutions. In most cases, pupils will find tables, lists and diagrams are needed to manage the data while exploring the different possibilities. In some cases, the number of possible solutions is endless and the added criteria of finding the 'smallest number' has been added overcome this dilemma. Problem 6 has at least two possible combinations to make 144.

Possible difficulties
- Difficulty in understanding place value
- Wanting to add, subtract or multiply rather than using place value or number sense
- Not using all the criteria
- Not using place value to solve the questions
- Not taking into consideration the starting page of each chapter

Extension
- Pupils could think up their own bookworm problems using different criteria.
- Change the criteria involving the number of pages in a book and the page number read to and solve the problems based on the new criteria.
- Pupils could make puzzle scroll cards where they use the existing question but change the numbers or context to make a new problem.

SAMPLE - NOT FOR SALE

BOOKWORMS

Nathan's book starts on page 1 and has 2232 pages. There are 18 chapters of equal length in the book and he has read up to page 1674.

1. How many pages are in each chapter?

.....

2. Nathan's favourite part of the story is on page 874. What chapter is it in?

.....

3. Nathan reads half a chapter each day.
   How long has he been reading the book?

.....

4. Nathan's favourite chapter is Chapter 11. What pages are in Chapter 11?

.....

5. The most exciting part was from page 1047 to page 1149. What chapters are these pages in?

.....

6. How many pages past the middle of the book has Nathan read to?

.....

7. Nathan's friend is also reading the same book. He has read 14 pages of Chapter 8. What page is he up to?

.....

8. How many more pages does Nathan's friend need to read to be where Nathan is?

.....

9. How many more pages does Nathan need to read to finish the book?

.....

10. How many more days will it take for Nathan to finish the book?

.....

SAMPLE - NOT FOR SALE

CALCULATOR PATTERNS

Use your calculator to help you solve the problems.

1. Choose a 3 digit number in which all 3 digits are different; e.g. 785.
   (a) Make all of the possible 2 digit numbers using these digits; e.g. 78, 75, 85, 87, 57, 58.

   _____

   _____

   (b) Find the sum of the numbers. _____
   (c) Divide this amount by the sum of the digits in your 3 digit number.
   (d) What answer did you get? _____
2. Try the same activity with some other numbers.

   _____

   _____

   _____

   _____
3. Why do you think this happens?

   _____

   _____
4. What happens if you do the same thing with a 4-digit number?

   _____

   _____

   _____

   _____
5. Why do you think this happens?

   _____

   _____
6. What do you think will happen if you make all of the 3-digit numbers?

   _____

   _____
7. What about the 4-digit numbers?

   _____

   _____

SAMPLE - NOT FOR SALE

PUZZLE SCROLLS 1

1. Two boxes of pencils and three boxes of pens cost €7.45, while 3 boxes of pencils and 2 boxes of pens cost €7.05.

   How much does 1 box of pens cost?

2. A box of beads is able to be equally shared among 2, 3, 4, 5 or 6 bags.

   What is the smallest number of beads that the box could have?

3. The numbers 1, 3, 5 and 7 are a set of four consecutive odd numbers.

   List the 4 consecutive odd numbers that have a sum of 360.

4. When 5 is added to one-third of a number, the result is half of the number.

   What is the number?

5. What two-digit numbers have a remainder of 4 when divided by 5 and a remainder of 2 when divided by 7?

   How many did you find?

6. Place addition symbols between the digits of the number 987 654 321 so that it totals 144.

   How many ways can you find?

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